Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2023
Deferred Examinations

Marking Scheme

Applied Mathematics

Higher Level

v. 2

Note to teachers and students on the marking schemes for the deferred examinations

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. However, it should be noted that the marking schemes for the deferred examinations may not necessarily be as detailed as the corresponding marking schemes for the main sitting of an examination, which serve to ensure consistency across a large team of examiners.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination, and the need to maintain consistency in standards between the main sitting and the deferred sitting and from year to year. In the case of the deferred examinations, this means that the level of detail may vary by question, as the marking scheme will only have been finalised for the questions attempted by the candidates who sat these examinations.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with a senior examiner when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes (whether for the main examinations or the deferred examinations) should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination concerned. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination will not necessarily be the same for the deferred sitting as for the main sitting or from one year to the next.

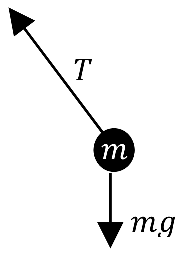
1ai

$AB = \begin{pmatrix} 2 & 7 & -2 \\ -9 & 10 & 8 \\ 7 & 1 & 2 \end{pmatrix}$ 10

1aii

e.g. the first element of BA is $6 \neq 2$. 5

1bi



5

1bii

$T \cos\theta = mg$ 5

$T \sin\theta = \frac{mv^2}{r} = \frac{mv^2}{l \sin\theta}$ 5

$mg \tan\theta = \frac{mv^2}{l \sin\theta}$ so $v = \sqrt{gl \sin\theta \tan\theta}$ 5

1biii

$T = \frac{2\pi r}{v}$ 5

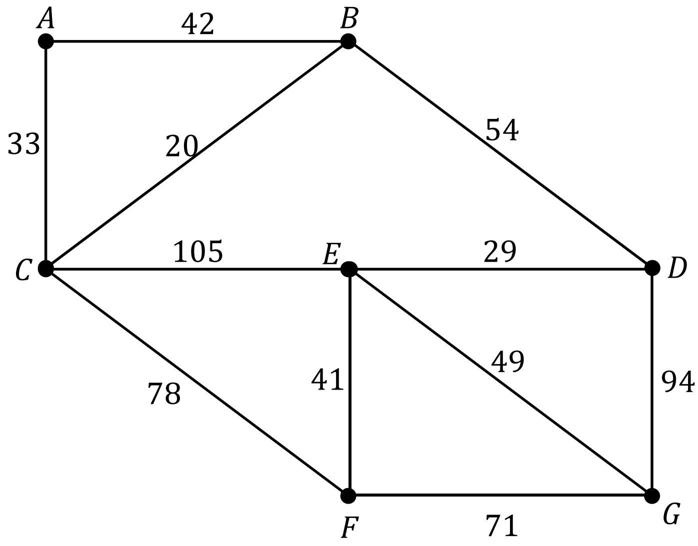
$T = \frac{2\pi l \sin\theta}{\sqrt{gl \sin\theta \tan\theta}} = 2\pi\sqrt{\frac{l \sin\theta}{g \tan\theta}} = 2\pi\sqrt{\frac{l \cos\theta}{g}}$ 5

1biv

$\cos\theta$ and 2π have no unit, l has units m and g has units m s^{-2}, so $\frac{l\cos\theta}{g}$ has units s^2.

Therefore T has units s. 5

2ai



10

2aii

A: $B(42)$, $C(33)$

C: ~~$B(53)$~~, ~~$E(138)$~~, $F(111)$

B: $D(96)$

D: $E(125)$, ~~$G(190)$~~

F: ~~$E(152)$~~, ~~$G(182)$~~

E: $G(174)$

Path $= A \to B \to D \to E \to G$ Time $= 174$ ms

Dijkstra's algorithm

15 [**0/6/9/12**]

2bi

$E_n = a^n E_0 + b\left(\frac{1-a^n}{1-a}\right)$ or $E_n = Pa^n + Q$ 5

$E_n = 1.15^n(101) - C\left(\frac{1.15^n-1}{0.15}\right)$ 5

2bii

$1.15^{21}(101) = C\left(\frac{1.15^{21}-1}{0.15}\right)$ 5

$C = 16$ to the nearest whole number 5

2biii

$\frac{dE}{dn} = 0.15E - C$ 5

3i

$G_n = P_n + I_n + S$ so $G_{n+2} = P_{n+2} + I_{n+2} + S = aG_{n+1} + b(P_{n+2} - P_{n+1}) + S$

i.e. $G_{n+2} = aG_{n+1} + b(aG_{n+1} - aG_n) + S = (a + ab)G_{n+1} - abG_n + S$ 5

$c = -a - ab = -a(1 + b) = -\frac{15}{16}\left(1 + \frac{3}{5}\right) = -\frac{3}{2}$ and $d = ab = \frac{15}{16} \cdot \frac{3}{5} = \frac{9}{16}$ 5

3ii

$G_{n+2} - \frac{3}{2}G_{n+1} + \frac{9}{16}G_n = 0$ has characteristic equation $x^2 - \frac{3}{2}x + \frac{9}{16} = 0$, i.e. $\left(x - \frac{3}{4}\right)^2 = 0$ 5

$\therefore G_n = m\left(\frac{3}{4}\right)^n + ln\left(\frac{3}{4}\right)^n$ 5

so $G_0 = 840 = m$ and $G_1 = 820 = 840\left(\frac{3}{4}\right) + l\left(\frac{3}{4}\right)$, i.e. $l = \frac{760}{3}$ 5

$G_6 = 420$ to the nearest billion euros 5

3iii

$G_{n+2} - \frac{3}{2}G_{n+1} + \frac{9}{16}G_n = 40$ has a particular solution of the form $f(n) = an + b$ 5

$f(n+2) - \frac{3}{2}f(n+1) + \frac{9}{16}f(n) = 40$ so $an + 2a + b - \frac{3}{2}an - \frac{3}{2}a - \frac{3}{2}b + \frac{9}{16}an + \frac{9}{16}b = 40$

i.e. $\frac{1}{2}a + \frac{1}{16}b + \frac{1}{16}an = 40$ for all n

$n = 0 \Rightarrow \frac{1}{2}a + \frac{1}{16}b = 40$ and $n = 1 \Rightarrow \frac{9}{16}a + \frac{1}{16}b = 40$

$a = 0$ and $b = 640$ 5

$G_n = m\left(\frac{3}{4}\right)^n + ln\left(\frac{3}{4}\right)^n + 640$, so $G_0 = 840 = m + 640$, i.e $m = 200$

and $G_1 = 820 = 200\left(\frac{3}{4}\right) + l\left(\frac{3}{4}\right) + 640$ i.e. $l = 40$ 5

$G_6 = 718$ to the nearest billion euros 5

4i

$\frac{dP}{dt} = rP$ so $\int \frac{dP}{P} = r\int dt$ i.e. $\ln P = rt + c = 0.08t + c$ 5

$P = 20$ when $t = 0$ so $c = \ln 20$ i.e. $\ln\frac{P}{20} = 0.08t$ so $P = 20e^{0.08t}$ 5

4ii

$t = 12 \Rightarrow P = 20e^{0.96} = 52$ insects to the nearest whole number 5

4iii

The population is unbounded, it will always increase. P will not stay small relative to K. 5

4iv

$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) = r\left(\frac{P(K-P)}{K}\right)$ so $\int \frac{KdP}{P(K-P)} = r\int dt$ i.e. $\int\left(\frac{1}{P} + \frac{1}{K-P}\right)dp = r\int dt$ 5

$\ln P - \ln(K-P) = rt + c'$ i.e. $\ln\frac{P}{K-P} = rt + c'$ 5

$P = 20$ when $t = 0$ so $c' = \ln\frac{20}{K-20}$ 5

i.e. $\ln\frac{P(K-20)}{20(K-P)} = rt$ so $\frac{P(K-20)}{20(K-P)} = e^{0.08t}$ i.e. $\frac{P}{K-P} = \frac{20}{K-20}e^{0.08t}$

$\therefore P = \frac{20K}{K-20}e^{0.08t} - \frac{20P}{K-20}e^{0.08t}$ so $P\left(1 + \frac{20e^{0.08t}}{K-20}\right) = \frac{20K}{K-20}e^{0.08t}$ i.e. $P\left(\frac{K-20+20e^{0.08t}}{K-20}\right) = \frac{20K}{K-20}e^{0.08t}$

i.e. $P = \frac{20Ke^{0.08t}}{K-20+20e^{0.08t}}$ 5

4v

$P = 39$ when $t = 12$ so $\frac{39}{K-39} = \frac{20}{K-20}e^{0.96} \Rightarrow \frac{K-20}{K-39} = \frac{20}{39}e^{0.96} \cong 1.339$

$\therefore K - 20 = 1.339(K-39)$ so $K = 95$ insects to the nearest whole number 5

4vi

K is the upper bound on the population. The size of the population tends to K over time. 5

5i

$v = u - gt$ so $0 = u - 9.8 \times 2$ i.e. $u = 19.6$ m s^{-1} 5

$s = ut + \frac{1}{2}at^2$ so $-24.5 = 19.6t - 4.9t^2$ 5

i.e. $t^2 - 4t - 5 = 0$ i.e. $(t - 5)(t + 1) = 0$

$t = 5$ s $(t \geq 0)$ 5

5ii

$\frac{dv}{dt} = -g - kv$ so $\int \frac{dv}{g+kv} = -\int dt$ 5

i.e. $\frac{1}{k}\ln|9.8 + kv| = -t + c$ 5

$v = 20$ when $t = 0$ so $c = \frac{1}{k}\ln|9.8 + 20k|$ 5

i.e. $\ln\frac{9.8+kv}{9.8+20k} = -kt$ so $9.8 + kv = (9.8 + 20k)e^{-kt}$

$v = 20e^{-kt} + \frac{9.8}{k}(e^{-kt} - 1)$ 5

5iii

$v = 0$ i.e. $20e^{-kt} = \frac{9.8}{k}(1 - e^{-kt})$

For $k = 0.1225$ this is $20e^{-0.1225t} = 80(1 - e^{-0.1225t})$ 5

i.e. $e^{-0.1225t} = 0.8$ i.e. $t = 1.82$ s 5

5iv

$\frac{dv}{dt} = g - kv$ 5

6ai

$v_B = v_C$ 5

i.e. $5.5 + 0.5t = 11 + 0.125t$ so $t = \frac{44}{3}$ s 5

$s_C - s_B = 11t + \frac{1}{2}0.125t^2 - 5.5t - \frac{1}{2}0.5t^2 = \frac{484}{3} + \frac{121}{9} - \frac{242}{3} - \frac{484}{9} = \frac{121}{3}$ m 5

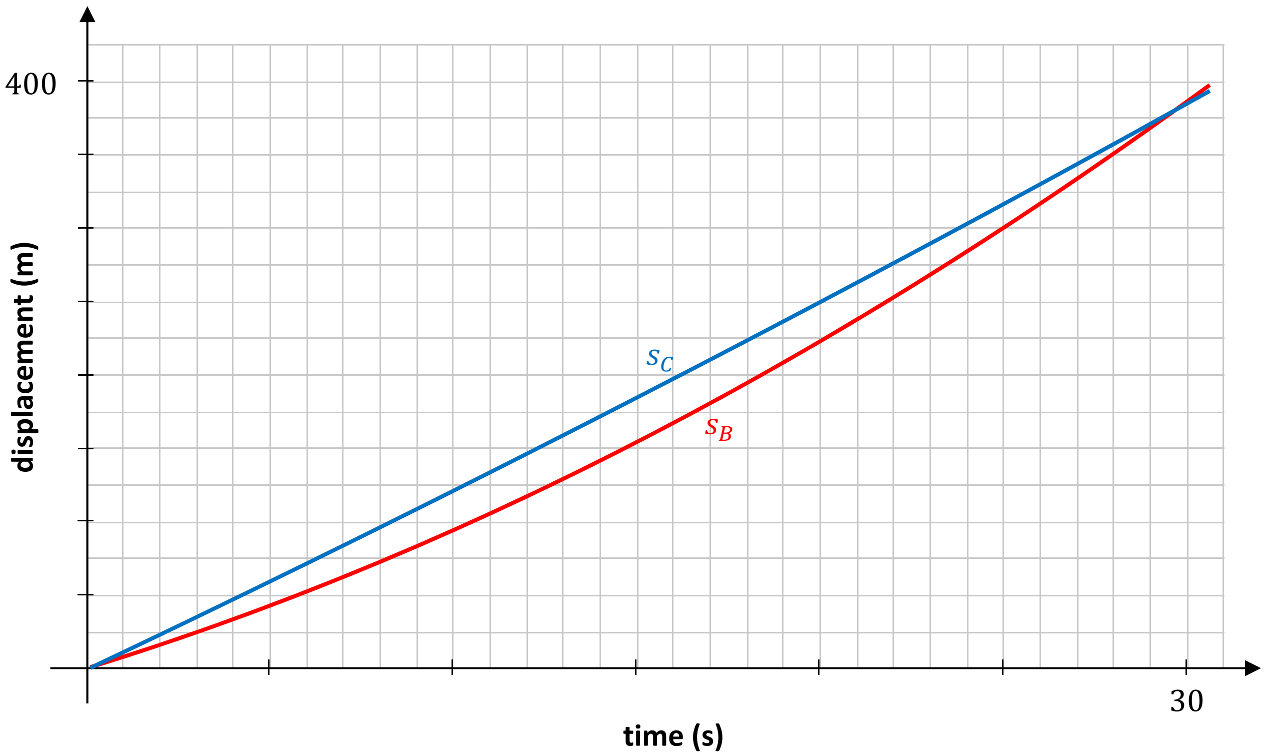
6aii

$s_B = s_C$ so $5.5t + \frac{1}{2}0.5t^2 = 11t + \frac{1}{2}0.125t^2$ i.e. $5.5t - 0.1875t^2 = 0$ i.e. $t(5.5 - 0.1875t) = 0$

$\therefore t = \frac{88}{3}$ s 5

$s_B = s_C = s = \frac{3388}{9}$ m 5

6aiii



5

6b

Stage	State	Action	Destination	Value (cost)
4	F	FY	Y	26*
	G	GY	Y	47*
	H	HY	Y	24*
3	D	DF	F	$28 + 26 = 54$*
		DH	H	$35 + 24 = 59$
	E	EF	F	$23 + 26 = 49$
		EG	G	$-5 + 47 = 42$*
2	A	AD	D	$7 + 54 = 61$*
	B	BD	D	$-9 + 54 = 45$*
		BE	E	$4 + 42 = 46$
	C	CE	E	$11 + 42 = 53$*
1	X	XA	A	$12 + 61 = 73$
		XB	B	$15 + 45 = 60$*
		XC	C	$8 + 53 = 61$

Path $= X \to B \to D \to F \to Y$ Minimum cost $= 60$

20 [**0/8/14/17**]

7i

		before impact (m s^{-1})	after impact (m s^{-1})	
P	$2m$	$u\cos\theta\,\vec{\imath} + u\sin\theta\,\vec{\jmath}$	$v_P\cos 2\theta\,\vec{\imath} + v_P\sin 2\theta\,\vec{\jmath}$	
Q	m	$0\vec{\imath} + 0\vec{\jmath}$	$v_Q\vec{\imath} + 0\vec{\jmath}$	5

$u\sin\theta = v_P\sin 2\theta = 2v_P\sin\theta\cos\theta$ so $u = 2v_P\cos\theta$

PCM $2m(u\cos\theta) + m(0) = 2m(v_P\cos 2\theta) + m(v_Q)$ 5

$2u\cos\theta = 2v_P\cos 2\theta + v_Q$

NEL $v_P\cos 2\theta - v_Q = -\frac{2}{3}u\cos\theta$ 5

$v_Q = v_P\cos 2\theta + \frac{2}{3}u\cos\theta$

$2u\cos\theta = 2v_P\cos 2\theta + v_P\cos 2\theta + \frac{2}{3}u\cos\theta$ so $\frac{4}{3}u\cos\theta = 3v_P\cos 2\theta$

$\frac{8}{3}v_P(\cos\theta)^2 = 3v_P\cos 2\theta$ so $8(\cos\theta)^2 = 9[(\cos\theta)^2 - (\sin\theta)^2]$ 5

$8 = 9 - 9(\tan\theta)^2$ so $9(\tan\theta)^2 = 1$ i.e. $\tan\theta = \frac{1}{3}$ as required 5

7ii

$v_Q = v_P\cos 2\theta + \frac{2}{3}u\cos\theta = \frac{u\cos 2\theta}{2\cos\theta} + \frac{2}{3}u\cos\theta$

$\frac{v_Q}{\cos\theta} = \frac{u}{2}(1 - (\tan\theta)^2) + \frac{2}{3}u = \frac{10u}{9}$ so $v_Q\sqrt{\frac{10}{9}} = \frac{10u}{9}$ i.e. $v_Q = \sqrt{\frac{10}{9}}u\vec{\imath} + 0\vec{\jmath}$ 5

7iii

$F = -kx$ and $W = \int F dx$ 5

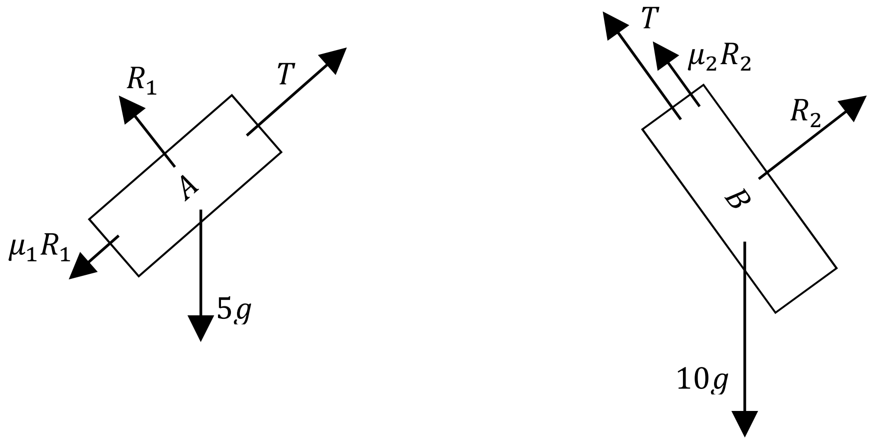
$\therefore W = \int kx dx = \frac{1}{2}kx^2 + c$ 5

$W = 0$ when $x = 0$, so $c = 0$ i.e. $W = \frac{1}{2}kx^2$ 5

7iv

$\frac{1}{2}m{v_Q}^2 = \frac{1}{2}kx^2$ so $x_{max} = \sqrt{\frac{10m}{9k}}u$ 5

8i



5, 5

8ii

$T - \frac{49}{8}\cos 40° - 49\sin 40° = 5a$ 5

$98\sin 55° - T - \frac{98}{5}\cos 55° = 10a$ 5

$15a = 98\sin 55° - \frac{98}{5}\cos 55° - \frac{49}{8}\cos 40° - 49\sin 40°$

i.e. $a = 2.19$ m s^{-2} and $T = 47.14$ N 5

8iii

$v^2 = u^2 + 2as$ i.e. $v^2 = 2 \times 2.19 \times 0.4$ so $v = 1.32$ m s^{-1} 5

8iv

$5f = -\frac{49}{8}\cos 40° - 49\sin 40°$ 5

$f = -7.24$ m s^{-2} 5

8v

$v^2 = u^2 + 2as$ i.e. $0^2 = 1.32^2 - 14.48s$ so $s = 0.12$ m 5

Total displacement $= 0.4 + 0.12 = 0.52$ m 5

9i

These are "dummy activities" that do not take any time but indicate the dependency of activities on prior activities. 5

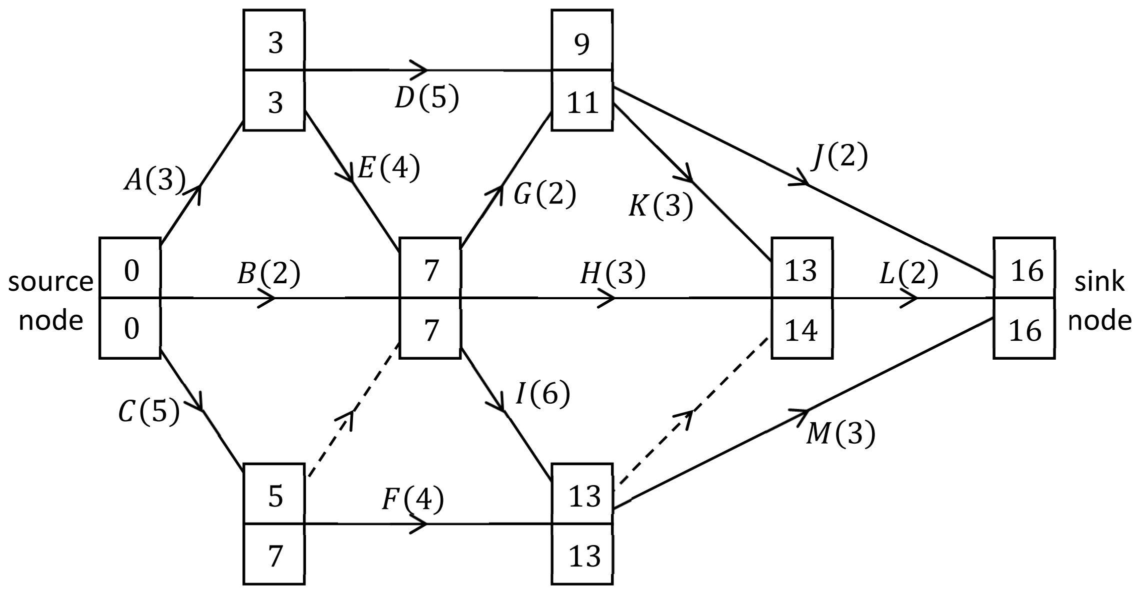
9ii

Activity	Depends directly on …	Activity	Depends directly on …
A	$-$	H	B, C, E
B	$-$	I	B, C, E
C	$-$	J	D, G
D	A	K	D, G
E	A	L	F, H, I, K
F	C	M	F, I
G	B, C, E		

10

–1 for each incorrect part

9iii



15 [**0/6/9/12**]

9iv

A, E, I, M 5

9v

16 hours 5

9vi

No effect, because it has a float of 3 hours, which is greater than $7 - 5 = 2$. 5

9vii

No effect on time taken, but activity J is now a critical activity and A, E, G, J becomes a new critical path. 5

10i

No friction, gravitational potential energy is converted into kinetic energy, object can be treated as a point mass, etc. 5

10ii

$3mg = \frac{1}{2}mv^2 + mg(1.5 + 1.5\cos\theta)$ i.e $v^2 = 3g(1 - \cos\theta)$ 5

$mg\cos\theta - R = \frac{mv^2}{r} = \frac{2mv^2}{3}$ 5

$R = 0$ 5

so $v^2 = \frac{3g\cos\theta}{2}$ i.e. $\frac{\cos\theta}{2} = 1 - \cos\theta$ so $\theta = \cos^{-1}\frac{2}{3}$ 5

10iii

$v^2 = \frac{3g\cos\theta}{2} = g$ so $v = \sqrt{9.8} = 3.13$ m s^{-1} at θ below the horizontal 5

10iv

$v = \sqrt{g}\cos\theta\,\vec{i} + \left(-\sqrt{g}\sin\theta - gt\right)\vec{j}$ 5

$1.5 + 1.5\cos\theta = \sqrt{g}\sin\theta\, t + \frac{g}{2}t^2$ 5

i.e. $gt^2 + \sqrt{5g}t - 5 = 0$ sp $t = \frac{-\sqrt{5g}\pm\sqrt{25g}}{2g} = 0.44$ s ($t > 0$) 5

10v

$D = \frac{2\sqrt{g}}{3}t + 1.5\sin\theta = 2.04$ m 5